Settling the active-passive debate empirically

What evidence is needed to detect outperformance?

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Introduction

Past performance $\rightarrow$ "track record" of fund manager.

Used to attract customers and justify fees.

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Industry consensus $\sim$ 5 years.

For sake of transparency, let's do the math.

Model

Standard financial model for asset value $x(t)$ is geometric Brownian motion (GBM).

Relative changes in $x(t)$ are normally distributed:

$$\frac{dx}{x} = \mu\, dt + \sigma\sqrt{dt}\, Z.$$

Drift μ, volatility σ, standard normal $Z \sim N(0, 1)$.

Model

After time t, $x(t)$ has grown exponentially at growth rate

$$g(t) = \underbrace{\mu - \frac{\sigma^2}{2}}_{\text{'skill'}} + \underbrace{\frac{\sigma Z}{\sqrt{t}}}_{\text{'luck'}}.$$

$t \to 0$, luck dominates skill: $g(t) \sim \frac{\sigma Z}{\sqrt{t}}$.

$t \to \infty$, skill dominates luck: $g(t) \sim \mu - \frac{\sigma^2}{2} \equiv \bar{g}$.

Active *vs* passive

Invest £1 in active fund and passive benchmark at time $t = 0$.

At time t, have $£x_A(t)$ in fund and $£x_P(t)$ in benchmark.

Parameters: μ_A, σ_A for fund; μ_P, σ_P for benchmark.

Assume fluctuations uncorrelated.

Active *vs* passive

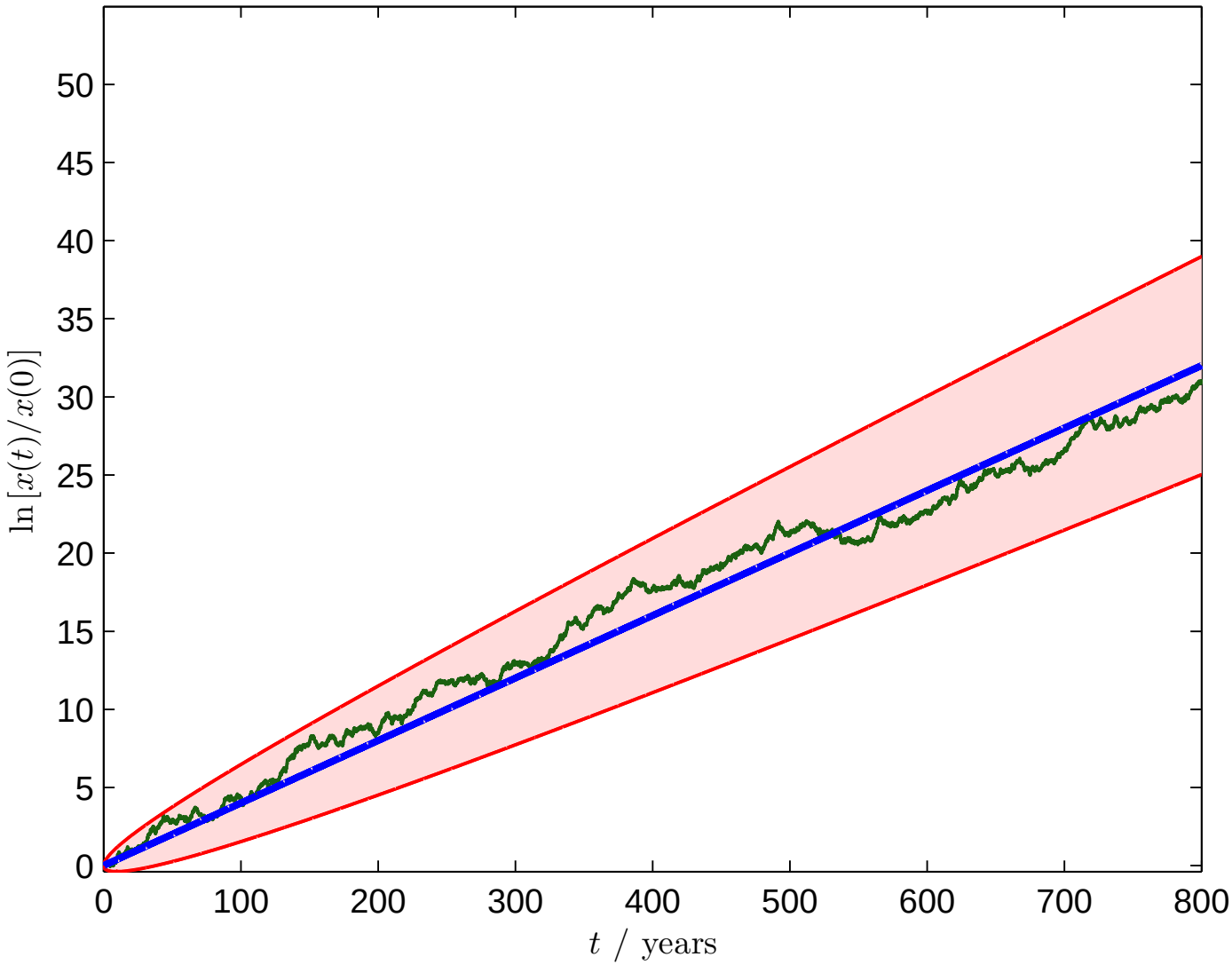
Long-run exponential growth rates:

$$\begin{aligned} \bar{g}_P &= \mu_P - \frac{{\sigma_P}^2}{2} \\ \bar{g}_A &= \mu_A - \frac{{\sigma_A}^2}{2} = \bar{g}_P + \alpha. \end{aligned}$$

α is outperformance of active relative to passive investment.

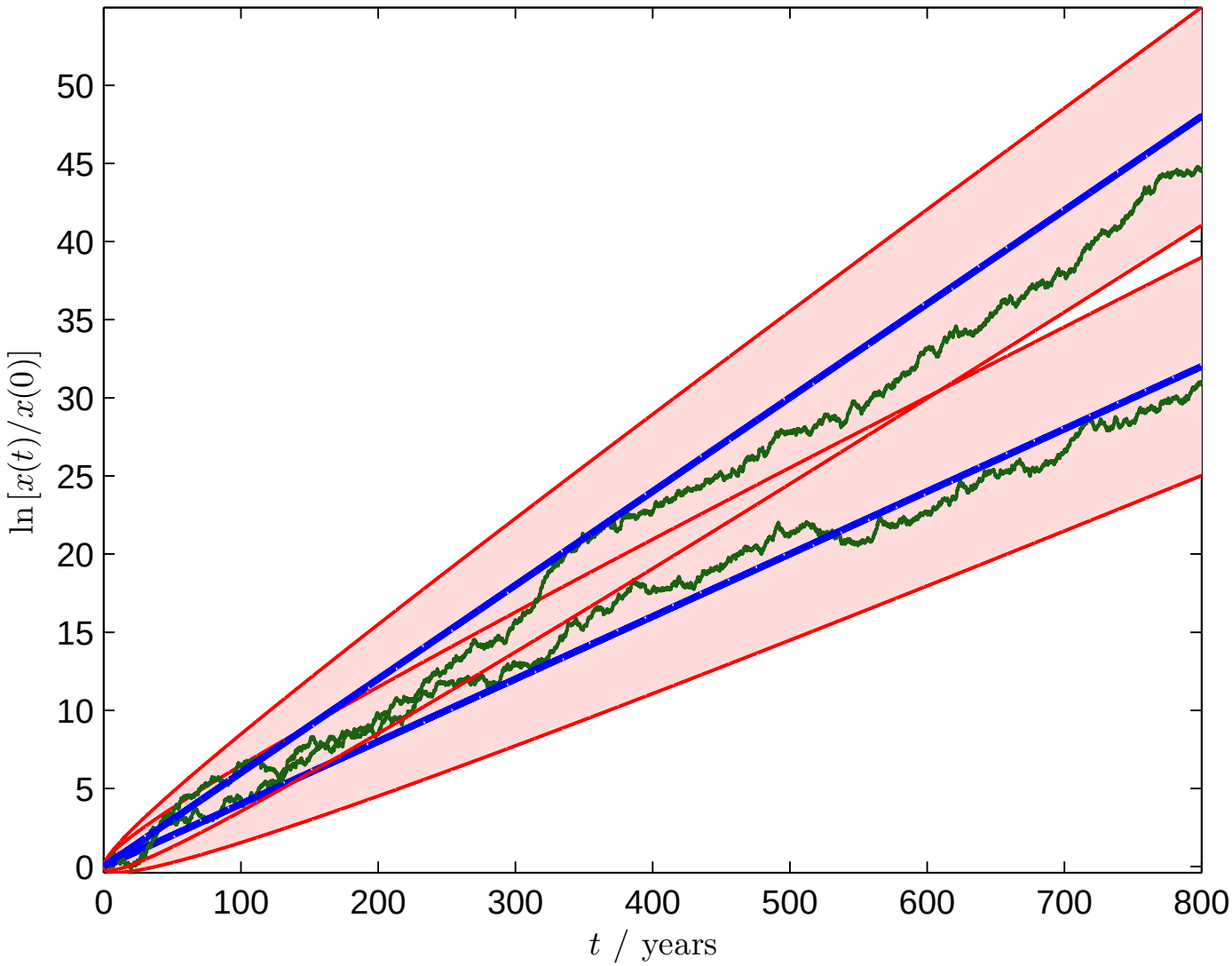
Active *vs* passive

Passive: $\bar{g}_P = 4\%\ pa$; $\sigma_P = 15\%\ pa$.



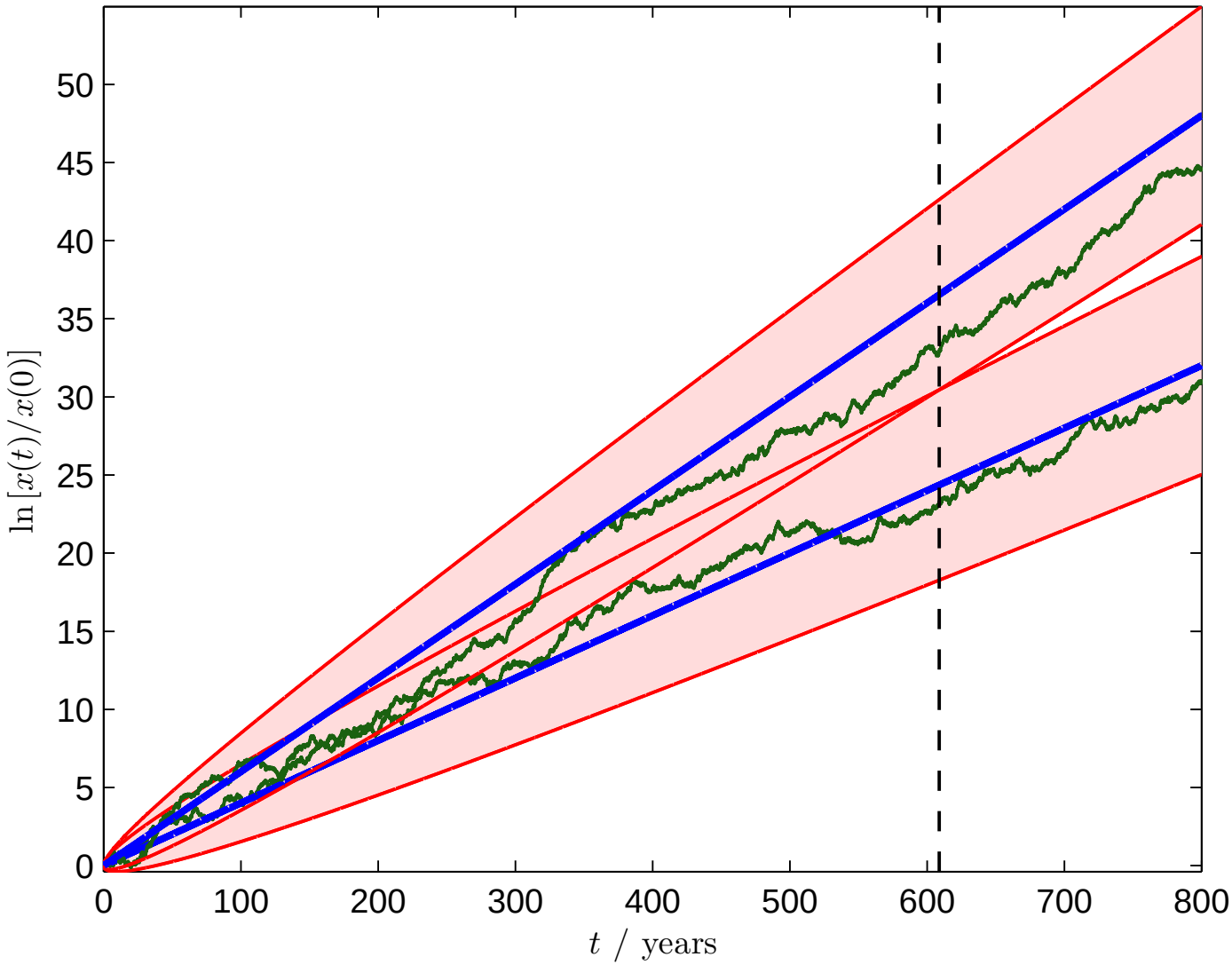
Active *vs* passive

Active: $\alpha = 2\%$ *pa*; $\sigma_A = \sigma_P$.



Active *vs* passive

Separation time: $t \approx 600$ years.



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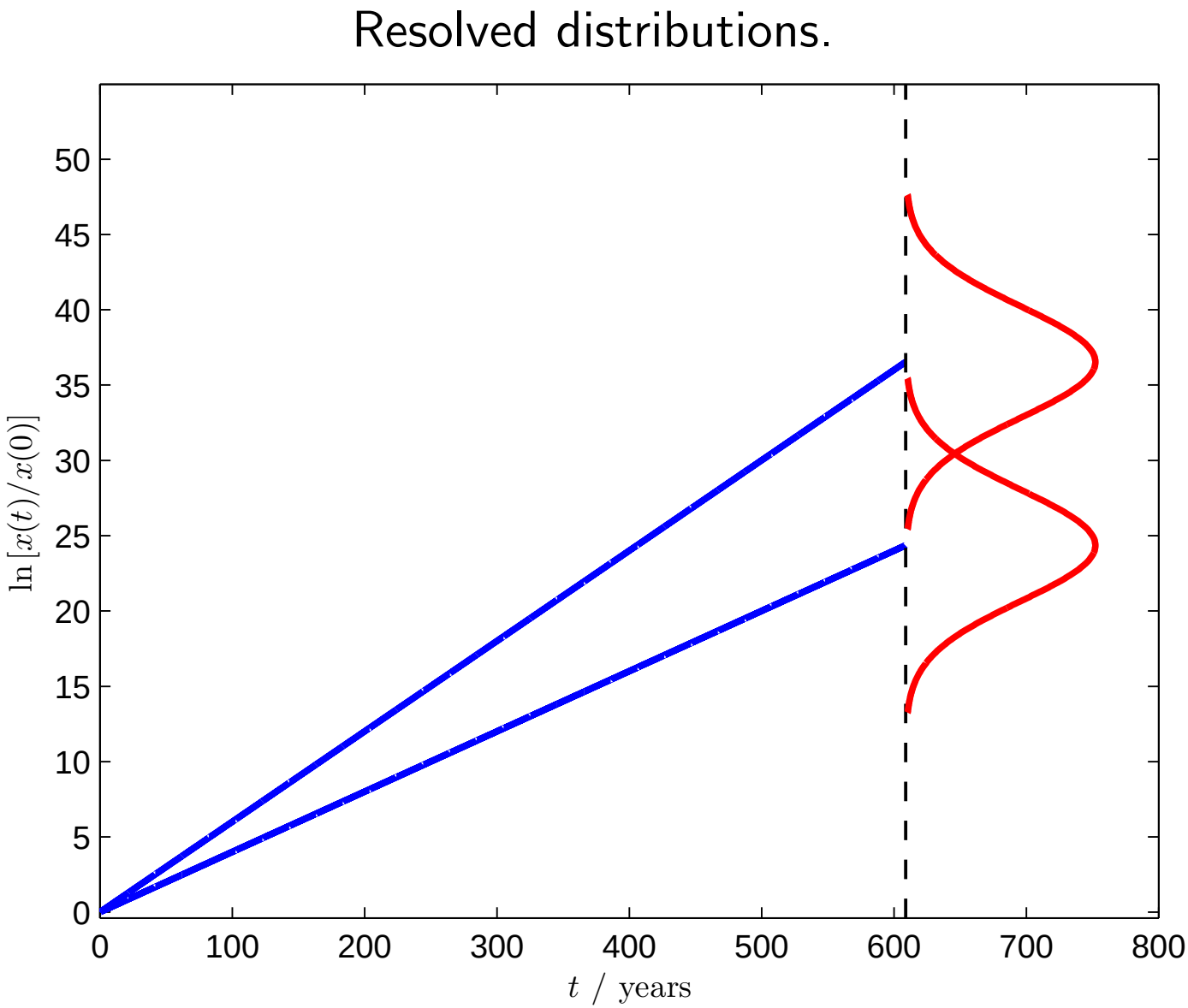


Detection time

Question

Suppose we observe outperformance α_T after time T.

How large must T be for confidence that α_T was due to skill?

Detection time

Answer

Null hypothesis: active and passive equally good ($\alpha = 0$).

Reject null hypothesis at **confidence level** c (*e.g.* 95%).

Find T that gives probability $1 - c$ of observing at least α_T under the null hypothesis.

In other words, unlikely to observe α_T after T by chance alone if active $=$ passive.

Detection time

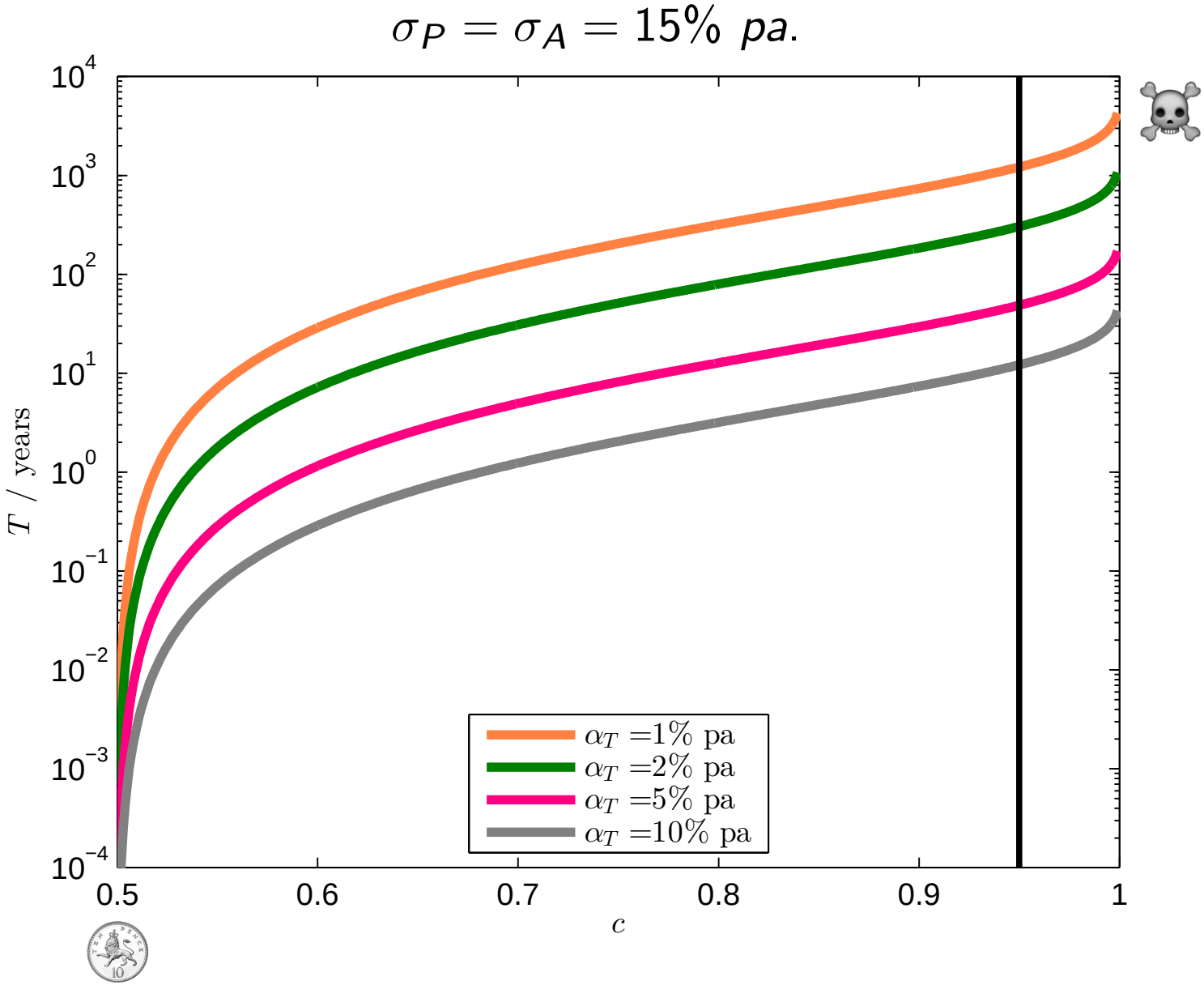
Some simple maths (normal distribution) gives us

$$T = \frac{{\sigma_P}^2 + {\sigma_A}^2}{{\alpha_T}^2} \left[\Phi^{-1}(c)\right]^2$$

where $\Phi(y)$ is the standard normal cdf.

Plot T against c for different values of α_T.

Detection time

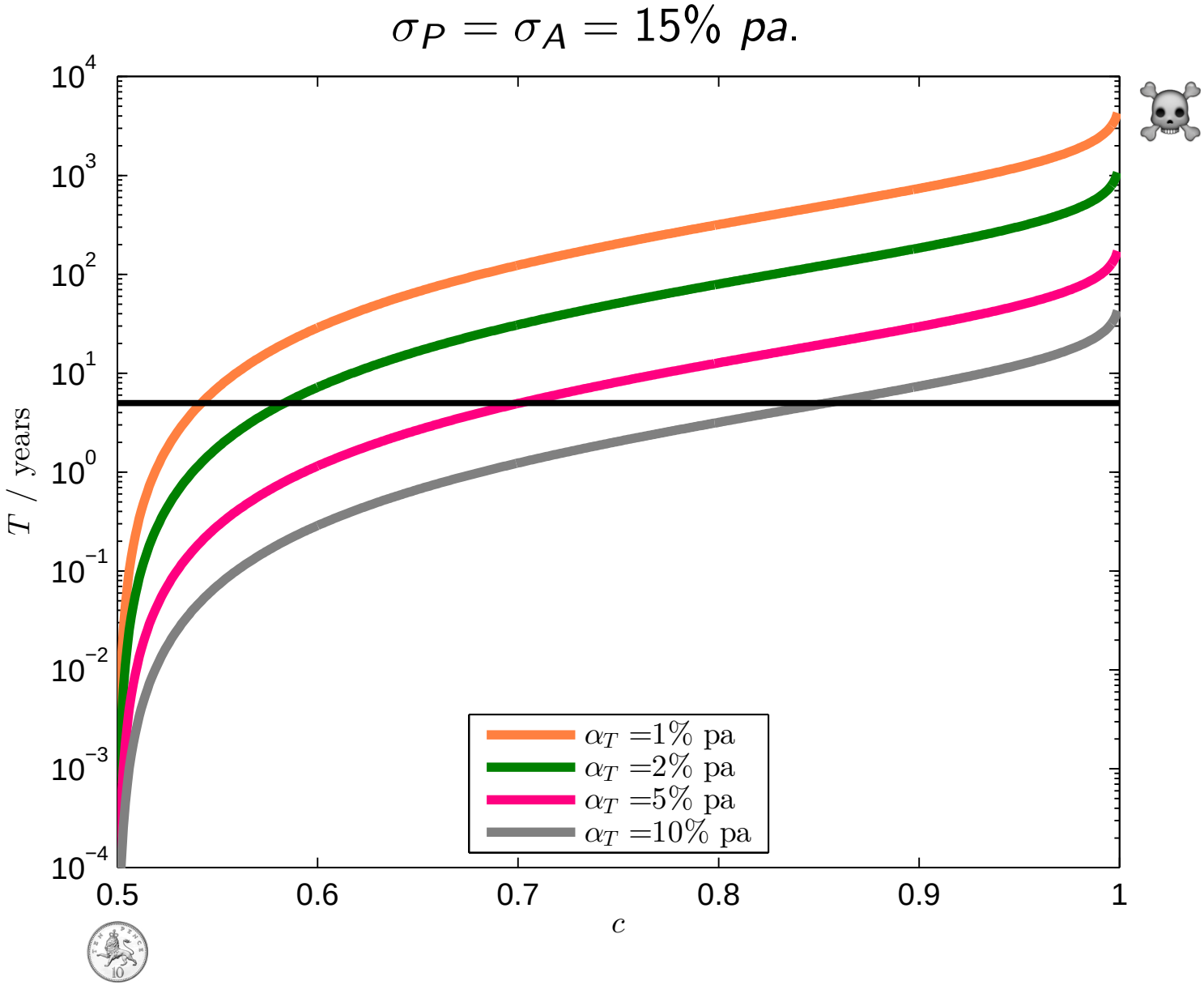


Detection time

Time to be 95% certain past performance due to skill:

α_T / *pa*	T / years
1%	1200
2%	300
5%	49
10%	12
16%	5

Detection time



Detection time

Confidence level after standard "track record" of 5 years:

α_T / *pa*	*c*
1%	54%
2%	58%
5%	70%
10%	85%
16%	95%

For $\alpha_T = 1\%$ or 2%, might as well flip a coin.

Correlations

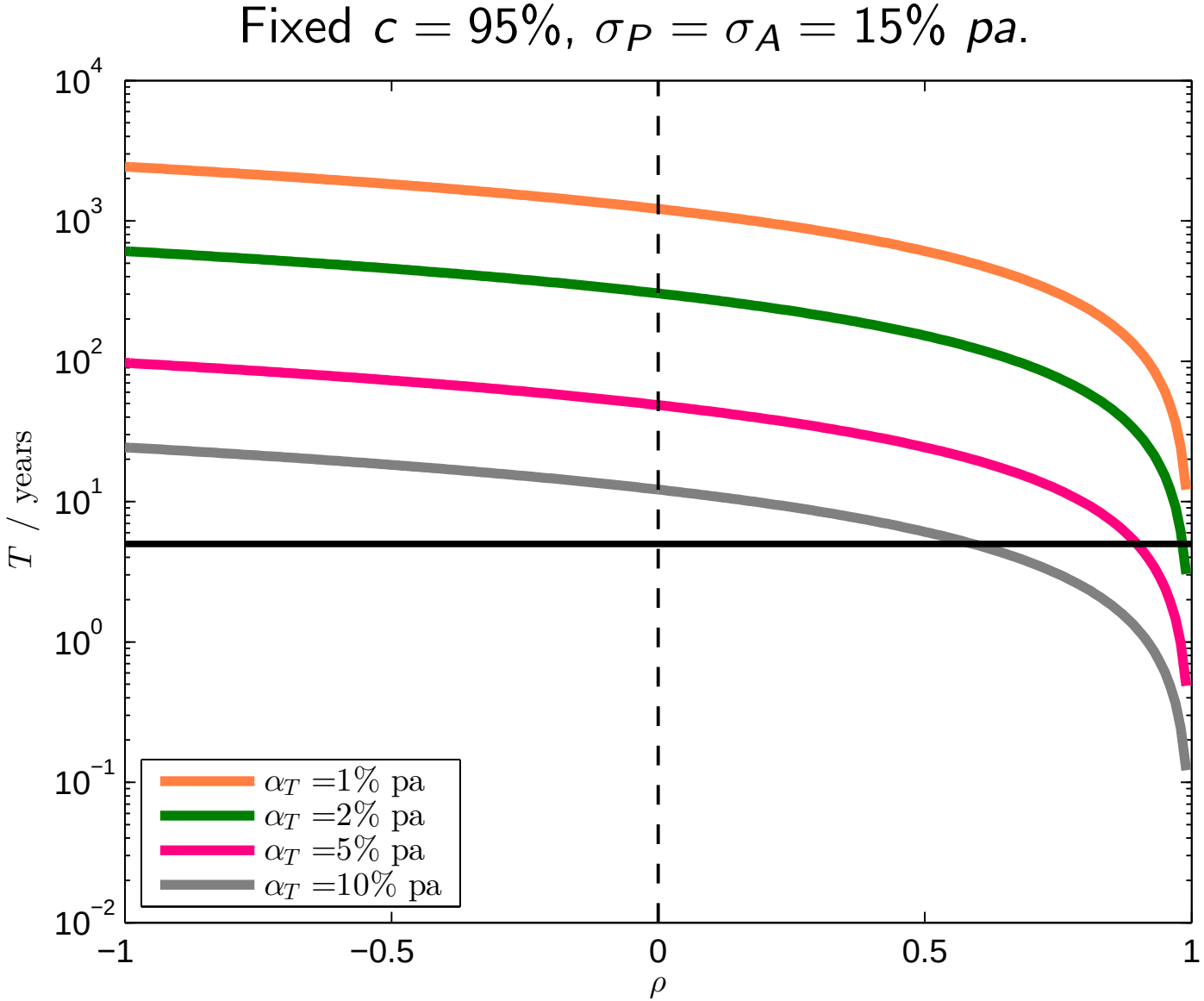
Independence assumption unrealistic – most funds are strongly correlated with benchmark (invest in same assets).

When fluctuations are correlated, observed outperformance is significant more quickly.

With perfect correlation, all outperformance is due to skill.

Include correlation coefficient ρ into calculation.

Correlations



Correlations

Find ρ required for $T = 5$ years at 95% confidence:

α_T / pa	ρ
1%	1
2%	0.99
5%	0.9
10%	0.59
16%	0

Correlations can rescue $T = 5$ years claim.

Evidence

What should empirical evidence of outperformance look like?

Four essential ingredients:

- observation time T;
- observed outperformance α_T;
- observed correlation coefficient ρ;
- desired confidence level c.

Any claim of "track record" should include these data to demonstrate performance distinguishable from luck.

Conclusions

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- $T = 5$ years claim is unscientific and often unwarranted.
- Deceitful to use short track records to advertise products or justify fees.
- For transparency past performance should be quoted with statistical confidence that it was skill not luck.
- Correlations can reduce detection time to human scales.

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